Pearson Edexcel

# Mark Scheme (Results)

## October 2020

Pearson Edexcel International A Level
In Further Pure Mathmatics F3
(WFM03/01)

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL IAL MATHEMATICS**

**General Instructions for Marking**

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

# General Principles for Further Pure Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

## Method mark for solving 3 term quadratic:

### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$ ,leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$ ,leading to x = …

### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

### 3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$ ,leading to x = …

## Method marks for differentiation and integration:

### 1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

### 2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **1(a)** | $4\sinh^3 x + 3\sinh x = 4\left(\frac{e^x - e^{-x}}{2}\right)^3 + 3\left(\frac{e^x - e^{-x}}{2}\right)$ <br> $= 4\left(\frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8}\right) + 3\left(\frac{e^x - e^{-x}}{2}\right)$ <br> Uses $\sinh x = \frac{e^x - e^{-x}}{2}$ on both sinh terms and attempts to cube the bracket <br> (min accepted is a linear x a quadratic bracket) | | M1 |
| | $= \frac{1}{2}e^{3x} - \frac{3}{2}e^x + \frac{3}{2}e^{-x} - \frac{1}{2}e^{-3x} + \frac{3}{2}e^x - \frac{3}{2}e^{-x}$ <br> $= \frac{e^{3x} - e^{-3x}}{2} = \sinh 3x$* | | A1* |
| | | | **(2)** |
| **(b)** | $\sinh 3x = 19\sinh x \Rightarrow 4\sinh^3 x + 3\sinh x = 19\sinh x$ <br> $\Rightarrow 4\sinh^3 x - 16\sinh x = 0$ <br> Uses the result from (a) and combines terms | | M1 |
| | ($\sinh x = 0$ or) $\sinh^2 x = 4$ | $\sinh^2 x = 4$ or $\sinh x = (\pm)2$ | A1 |
| | (0, 0) | States the origin as one intersection | B1 |
| | $\ln\left(2+\sqrt{5}\right)$ **and** $-\ln\left(2+\sqrt{5}\right)$ | Two correct non-zero $x$ values(allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$) | A1 |
| | $\left(\ln\left(2+\sqrt{5}\right), 38\right)$ **and** $\left(-\ln\left(2+\sqrt{5}\right), -38\right)$ | Two correct **points** (allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$) | A1 |
| | | | **(5)** |
| | **Alternative for (b) using exponentials** | | |
| | $\sinh 3x = 19\sinh x \Rightarrow \frac{e^{3x} - e^{-3x}}{2} = \frac{19\left(e^x - e^{-x}\right)}{2} \Rightarrow \ldots$ <br> Substitutes the correct exponential forms and collects terms to one side | | M1 |
| | $\Rightarrow e^{6x} - 19e^{4x} + 19e^{2x} - 1 = 0$ | Correct equation (or equivalent) | A1 |
| | (0, 0) | States the origin as one intersection | B1 |
| | $\frac{1}{2}\ln\left(9+4\sqrt{5}\right)$ **or** $\frac{1}{2}\ln\left(9-4\sqrt{5}\right)$ | Two correct non-zero $x$ values (oe) | A1 |
| | $\left(\frac{1}{2}\ln\left(9+4\sqrt{5}\right), 38\right)$ **and** $\left(\frac{1}{2}\ln\left(9-4\sqrt{5}\right), -38\right)$ | Two correct **points** (oe) | A1 |
| | | | **Total 7** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2(i)** | $3x^2+12x+24=3\left(x^2+4x+8\right)$ <br> $=3\left(\left(x+2\right)^2+4\right)$ | Obtains $3\left(\left(x+2\right)^2+...\right)$ or <br> $3\left(x+2\right)^2+...$ <br> Must include 3 now or later | M1 |
| | $3\left(\left(x+2\right)^2+4\right)$ or $3\left(x+2\right)^2+12$ | | A1 |
| | $\int \frac{1}{3x^2+12x+24}\,\mathrm{d}x=\frac{1}{3}\int \frac{1}{\left(x+2\right)^2+4}\,\mathrm{d}x=\frac{1}{6}\arctan\frac{x+2}{2}\left(+c\right)$ <br> M1: Use of arctan <br> A1: Fully correct expression (condone omission of $+c$) | | M1A1 |
| | | | **(4)** |
| **(ii)** | $27-6x-x^2=-\left(x^2+6x-27\right)$ <br> $=-\left(\left(x+3\right)^2-36\right)$ | Obtains $-\left(\left(x+3\right)^2+...\right)$ or <br> $-\left(x+3\right)^2+...$ | M1 |
| | $-\left(\left(x+3\right)^2-36\right)$ or $36-\left(x+3\right)^2$ | | A1 |
| | $\int \frac{1}{\sqrt{27-6x-x^2}}\,\mathrm{d}x=\int \frac{1}{\sqrt{36-\left(x+3\right)^2}}\,\mathrm{d}x=\arcsin\left(\frac{x+3}{6}\right)\left(+c\right)$ <br> (Or $=-\arccos\left(\frac{x+3}{6}\right)\left(+c\right)$) <br> M1: Use of arcsin (or – arccos) <br> A1: Fully correct expression (condone omission of $+c$) | | M1A1 |
| | | | **(4)** |
| | | | **Total 8** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3** | $\mathbf{M}=\begin{pmatrix}3 & -4 & k\\ 1 & -2 & k\\ 1 & -5 & 5\end{pmatrix}$ | | |
| **(a)** | $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\mathbf{M}=\begin{vmatrix}3-\lambda & -4 & k\\ 1 & -2-\lambda & k\\ 1 & -5 & 5-\lambda\end{vmatrix}=\begin{vmatrix}0 & -4 & k\\ 1 & -5 & k\\ 1 & -5 & 2\end{vmatrix}$<br>$(0)+4[2-k]+k[-5+5]$<br>Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert$ using $\lambda=3$ | | M1 |
| | $(0)+4[2-k]+k[-5+5]=0\Rightarrow k=...$<br>Uses $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ and solves for $k$ | | M1 |
| | $k=2$ | Cao | A1 |
| | | | **(3)** |
| **(b)** | $(3-\lambda)[(\lambda+2)(\lambda-5)+10]+4(5-\lambda-2)+2(-5+2+\lambda)=0$<br>Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ using their value of $k$ | | M1 |
| | $\Rightarrow(3-\lambda)[(\lambda+2)(\lambda-5)+12]=0$<br>$(\lambda+2)(\lambda-5)+12\Rightarrow\lambda^2-3\lambda+2=0\Rightarrow(\lambda-2)(\lambda-1)=0\Rightarrow\lambda=...$<br>Uses $\lambda=3$as a factor to obtain and solve a 3TQ to find the other eigenvalues<br>(Alternatively may use calculator to solve $\lambda^3-6\lambda^2+11\lambda-6=0$) | | M1 |
| | $\lambda=1,\ 2$ | Correct values | A1 |
| | | | **(3)** |
| **(c)** | $\begin{pmatrix}3 & -4 & 2\\ 1 & -2 & 2\\ 1 & -5 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=3\begin{pmatrix}x\\ y\\ z\end{pmatrix}\Rightarrow\begin{matrix}3x-4y+2z=3x\\ x-2y+2z=3y\\ x-5y+5z=3z\end{matrix}$ | Uses the eigenvalue 3 and their $k$ to form at least 2 equations in $x$, $y$ and $z$ | M1 |
| | $\alpha\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ (α a constant) | Any correct eigenvector. Allow any constant multiple of **i** + **j** + 2**k** | A1 |
| | $\frac{1}{\sqrt{6}}\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ | Correct normalised vector | A1 |
| | | | **(3)** |
| | | | **Total 9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **4.** | $I_n = \int x^n \cos x \, \mathrm{d}x$ | | |
| **(a)** | $\int x^n \cos x \, \mathrm{d}x = x^n \sin x - \int n x^{n-1} \sin x \, \mathrm{d}x$<br>M1: Parts in the correct direction<br>A1: Correct expression | | M1A1 |
| | $= x^n \sin x - \left\{ -n x^{n-1} \cos x + \int n(n-1) x^{n-2} \cos x \, \mathrm{d}x \right\}$<br>Uses integration by parts again (**dependent on the first M**) | | dM1 |
| | $= x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT** | | | |
| | $I_n = \int x^n \cos x \, \mathrm{d}x = \int x^{n-1} (x \cos x) \, \mathrm{d}x$ | | |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-2} (x \sin x + \cos x) \, \mathrm{d}x$<br>M1: Parts in the correct direction<br>A1: Correct expression | | M1A1 |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-1} \sin x \mathrm{d}x - (n-1) I_{n-2}$ | | |
| | $= x^n \sin x + x^{n-1} \cos x - (n-1) \left\{ -x^{n-1} \cos x + (n-1) I_{n-2} \right\} - (n-1) I_{n-2}$<br>Uses integration by parts again (**dependent on the first M**) | | dM1 |
| | $= x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}$ *<br>Fully correct proof with no errors | | A1* |
| | | | |
| **(b)** | $I_0 = \sin x \ \ (+k)$ | | B1 |
| | $I_4 = x^4 \sin x + 4x^3 \cos x - 12 I_2$ | Applies the reduction formula once for $I_4$ or $I_2$ | M1 |
| | $= x^4 \sin x + 4x^3 \cos x - 12 \left( x^2 \sin x + 2x \cos x - 2 I_0 \right)$<br>Applies the reduction formula again and obtains an expression for $I_4$ which can include $I_0$ but not $I_2$ | | M1 |
| | $= \left( x^4 - 12x^2 + 24 \right) \sin x + \left( 4x^3 - 24x \right) \cos x + c$<br>Award A1 for either bracket and A1 for the other<br>If the answer is not factorised but is otherwise correct, award A1A0 | | A1A1 |
| | | | **(5)** |
| | | | **Total 9** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5** | $\frac{x^2}{25}-\frac{y^2}{4}=1 \quad y=mx+c$ | | |
| **(a)** | $\frac{x^2}{25}-\frac{(mx+c)^2}{4}=1 \Rightarrow 4x^2-25\left(m^2x^2+2cmx+c^2\right)=100$<br>Substitutes to obtain a quadratic in $x$ and eliminates fractions | | M1 |
| | $4x^2-25\left(m^2x^2+2cmx+c^2\right)=100$<br>$\left(\Rightarrow \left(25m^2-4\right)x^2+50cmx+25c^2+100=0\right)$<br>Correct 3TQ | | A1 |
| | $"b^2=4ac" \Rightarrow (50cm)^2=4\left(25m^2-4\right)\left(25c^2+100\right)$<br>Uses '$b^2=4ac$' or equivalent | | M1 |
| | $2500c^2m^2=2500c^2m^2+10000m^2-400c^2-1600$<br>$10000m^2=400c^2+1600$<br>$25m^2=c^2+4$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT 1** | Using hyperbolic parameters: | | |
| | $x=5\cosh t, y=2\sinh t \Rightarrow \frac{dy}{dx}=\frac{2\cosh t}{5\sinh t}$ | | |
| | $\frac{2\cosh t}{5\sinh t}(x-5\cosh t)=y-2\sinh t$<br>M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed) | | M1A1 |
| | $y=\frac{2\cosh t}{5\sinh t}x-\frac{2\cosh^2 t-25\sinh^2 t}{\sinh t}$ | | |
| | $25m^2=\frac{4\cosh^2 t}{\sinh^2 t},\ 4+c^2=4+\frac{4}{\sinh^2 t}=\frac{4\left(\sinh^2 t+1\right)}{\sinh^2 t}=\frac{4\cosh^2 t}{\sinh^2 t}$<br>Extracts $25m^2$ and $4+c^2$ from their equation | | M1 |
| | $\therefore 25m^2=4+c^2$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |
| **ALT 2** | Using trigonometric parameters: | | |
| | $x=5\sec t, y=2\tan t \Rightarrow \frac{dy}{dx}=\frac{2\sec t}{5\tan t}$ | | |
| | $\frac{2\sec t}{5\tan t}(x-5\sec t)=y-2\tan t$<br>M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed) | | M1A1 |
| | $y=\frac{2\sec t}{5\tan t}x+\frac{2\tan^2 t-2\sec^2 t}{\tan t}$ | | |
| | $25m^2=\frac{4\sec^2 t}{\tan^2 t}=\frac{4}{\sin^2 t} \quad 4+c^2=4\left(1+\frac{1}{\tan^2 t}\right)=4\left(\frac{\sin^2 t+\cos^2 t}{\sin^2 t}\right)=\frac{4}{\sin^2 t}$ Extracts<br>$25m^2$ and $4+c^2$ from their equation | | M1 |
| | $\therefore 25m^2=4+c^2$ *<br>Fully correct proof with no errors | | A1* |
| | | | **(4)** |

| | | | |
|---|---|---|---|
| **(b)** | $25m^2 = c^2 + 4$ and $2 = m + c$<br>$25m^2 = (2-m)^2 + 4$ or $25(2-c)^2 = c^2 + 4$<br>Uses the given hyperbola and the straight line with the result from (a) to obtain an equation in $m$ or $c$ | | M1 |
| | $24m^2 + 4m - 8 = 0$<br>or<br>$24c^2 - 100c + 96 = 0$ | Correct 3TQ in $m$ or $c$ | A1 |
| | $24m^2 + 4m - 8 = 0 \Rightarrow m = \frac{1}{2}, -\frac{2}{3}$<br>Or<br>$24c^2 - 100c + 96 = 0 \Rightarrow c = \frac{3}{2}, \frac{8}{3}$ | Solves their 3TQ in $m$ or $c$ | M1 |
| | $y = \frac{1}{2}x + \frac{3}{2}$ **or** $y = -\frac{2}{3}x + \frac{8}{3}$ | One correct tangent | A1 |
| | $y = \frac{1}{2}x + \frac{3}{2}$ **and** $y = -\frac{2}{3}x + \frac{8}{3}$ | Both correct tangents | A1 |
| | | | **(5)** |
| **(c)** | $m = \frac{1}{2},\ c = \frac{3}{2} \Rightarrow \frac{9}{4}x^2 + \frac{75}{2}x + \frac{625}{4} = 0 \Rightarrow x = \ldots$<br>**or**<br>$m = -\frac{2}{3},\ c = \frac{8}{3} \Rightarrow \frac{64}{9}x^2 - \frac{800}{9}x + \frac{2500}{9} = 0 \Rightarrow x = \ldots$<br>Uses one of their $m$ and $c$ pairs and solves for $x$ | | M1 |
| | $x = -\frac{25}{3}, y = -\frac{8}{3}$ **or** $x = \frac{25}{4}, y = -\frac{3}{2}$ | One correct point | A1 |
| | $x = -\frac{25}{3}, y = -\frac{8}{3}$ **and** $x = \frac{25}{4}, y = -\frac{3}{2}$ | Both correct points | A1 |
| | | | **(3)** |
| | | | **Total 12** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **6(a)** | $\mathbf{A}=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}$ | | |
| | $\lvert\mathbf{A}\rvert = a-2+a-1+2-1\left(=2a-2\right)$ | Correct determinant in any form | B1 |
| | $\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}\to\begin{pmatrix}a-2 & a-1 & 1\\ -a-2 & a-1 & 3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}$<br>Applies the correct method to reach at least a matrix of cofactors<br>2 correct rows or 2 correct columns needed | | M1 |
| | $\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$<br>Correct transpose of cofactors | | A1 |
| | $\mathbf{A}^{-1}=\dfrac{1}{2a-2}\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$ | Correct inverse | A1 |
| | | | **(4)** |
| **(b)** | $a=4\Rightarrow\mathbf{A}^{-1}=\dfrac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $=\dfrac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}12-6\lambda\\ 4+2\lambda\\ 6+3\lambda\end{pmatrix}=\ldots$ | Attempt to multiply the parametric form of $l_2$ by their inverse | M1 |
| | $=\begin{pmatrix}6-\lambda\\ -4+4\lambda\\ 2-\lambda\end{pmatrix}$ | Correct parametric form | A1 |
| | $\mathbf{r}=\begin{pmatrix}6\\ -4\\ 2\end{pmatrix}+\lambda\begin{pmatrix}-1\\ 4\\ -1\end{pmatrix}$ | Correct equation (allow equivalent forms) but if given as $l=\ldots$ award A0 | A1 |
| | | | **(4)** |
| | | | **Total 8** |
| | | | |

| | | | |
|---|---|---|---|
| | **Alternatives for (b)** | | |
| | | | |
| **(i)** | $a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$<br>$\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} -6 \\ 24 \\ -6 \end{pmatrix}$ | Attempt $\mathbf{A}^{-1}$(point on $l_2$)<br>**and** $\mathbf{A}^{-1}$(direction of $l_2$)<br><br>**Both** correct (NB No ft) | M1<br><br>A1 |
| | $\mathbf{r}=\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$ | Correct equation (allow equivalent forms) but if given as $l$ = ... award A0 | A1 |
| | | | **(4)** |
| **(ii)** | $a=4 \Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ | Correct inverse (follow through their matrix from (a)) | B1ft |
| | $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$<br>$\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \\ 9 \end{pmatrix}=\frac{1}{6}\begin{pmatrix} 30 \\ 0 \\ 6 \end{pmatrix}$ | Attempt $\mathbf{A}^{-1}$(point on $l_2$) for 2 points<br><br>**Both** correct (NB No ft) | M1<br><br>A1 |
| | $\mathbf{r}=\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}+\lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$ | Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l$ = ... award A0 | A1 |
| | | | **(4)** |
| | | | |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7** | $x=\cosh t+t,\quad y=\cosh t-t$ | | |
| **(a)** | $\frac{\mathrm{d}x}{\mathrm{d}t}=\sinh t+1,\quad \frac{\mathrm{d}y}{\mathrm{d}t}=\sinh t-1$ | Correct derivatives | B1 |
| | $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=\sinh^2 t+2\sinh t+1+\sinh^2 t-2\sinh t+1$ $=2\sinh^2 t+2$ M1: Squares correctly, cancels and collects terms | | M1 |
| | $=2\left(1+\sinh^2 t\right)=2\cosh^2 t*$ | Uses $\cosh^2 t=1+\sinh^2 t$ to complete the proof with no errors | A1* |
| | | | **(3)** |
| **(b)** | $S=2\pi\int y\,\mathrm{d}s=2\pi\int(\cosh t-t)\sqrt{2}\cosh t\,\mathrm{d}t$ | Uses $S=2\pi\int y\,\mathrm{d}s$ with the given $y$ and the result from part (a) | M1 |
| | $=2\sqrt{2}\pi\int_0^{\ln 3}\left(\cosh^2 t-t\cosh t\right)\mathrm{d}t*$ | Correct proof with no errors | A1* |
| | | | **(2)** |
| **(c)** | $\int\cosh^2 t\,\mathrm{d}t=\int\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t\,\mathrm{d}t$ | Uses $\cosh^2 t=\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t$ | M1 |
| | $\int t\cosh t\,\mathrm{d}t=t\sinh t-\int\sinh t\,\mathrm{d}t$ | Attempts integration by parts the right way round on $t\cosh t$ | M1 |
| | | Correct expression | A1 |
| | $S=\left(2\sqrt{2}\pi\right)\int\left(\cosh^2 t-t\cosh t\right)\mathrm{d}t=\left(2\sqrt{2}\pi\right)\left[\frac{1}{2}t+\frac{1}{4}\sinh 2t-t\sinh t+\cosh t\right]$ A1: 2 correct terms A1: All correct | | A1A1 |
| | $(S=)2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3+\frac{10}{9}-\frac{4}{3}\ln 3+\frac{5}{3}\right)-(1)\right\}$ dM1: Correct use of limits 0 and ln3 depends on both preceding M marks | | dM1 |
| | $S=\frac{1}{9}\sqrt{2}\pi(32-15\ln 3)$ | cao | A1 **(7)** |
| | | | **Total 12** |
| | **Alternative for (c)** | | |
| | $\int\cosh^2 t\,\mathrm{d}t=\int\left(\frac{\mathrm{e}^t+\mathrm{e}^{-t}}{2}\right)^2\mathrm{d}t$ $=\frac{1}{4}\int\left(\mathrm{e}^{2t}+2+\mathrm{e}^{-2t}\right)\mathrm{d}t$ | Substitutes the exponential form and attempts to square | M1 |
| | $\int t\cosh t\,\mathrm{d}t=\frac{1}{2}\int t\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)\mathrm{d}t$ $=\frac{1}{2}t\mathrm{e}^t-\frac{1}{2}\int t\mathrm{e}^t\mathrm{d}t-\left\{\frac{1}{2}t\mathrm{e}^{-t}-\frac{1}{2}\int\mathrm{e}^{-t}\mathrm{d}t\right\}$ | Substitutes the exponential form and attempts integration by parts the right way round Correct expression | M1 A1 |
| | $(S=)\left(2\sqrt{2}\pi\right)\left\{\frac{1}{4}\left(\frac{1}{2}\mathrm{e}^{2t}+2t-\frac{1}{2}\mathrm{e}^{-2t}\right)-\frac{1}{2}t\mathrm{e}^t+\frac{1}{2}\mathrm{e}^t+\frac{1}{2}t\mathrm{e}^{-t}-\frac{1}{2}\mathrm{e}^{-t}\right\}$ A1: either integral correct A1: other integral correct but both must be in a complete expression for $S$ | | A1A1 |
| | Depends on both M marks above | Correct use of limits 0 and ln3 | dM1 |
| | $S=\frac{1}{9}\sqrt{2}\pi(32-15\ln 3)$ | cao | A1 |

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| | **Alternative for the first 3 marks of (c)** | | |
| | $= 2\sqrt{2}\pi\int \left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$<br>$= 2\sqrt{2}\pi\int \cosh t\left(\cosh t - t\right)\mathrm{d}t$ | | |
| | $2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right] - \int \sinh t\left(\sinh t - 1\right)\mathrm{d}t\right)$ | | |
| | $2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right] - \left[\cosh t\left(\sinh t - 1\right)\right] + \int \cosh^2 t\mathrm{d}t\right)$<br>M1 (2nd on e-PEN): Use parts twice A1 Correct expression | | M1A1 |
| | $\int \cosh^2 t\,\mathrm{d}t = \int \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t\,\mathrm{d}t$ | Uses $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ | M1 (1st on e-PEN) |
| | Rest as main scheme | | |
| | | | |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8(a)** | $\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$ | Attempt vector product between normal vectors | M1 |
| | $=\begin{pmatrix}-4\\7\\13\end{pmatrix}$ | Correct vector | A1 |
| | $x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$<br>$\Rightarrow y=-\frac{1}{4}, z=\frac{13}{4}$<br>or<br>$y=0\Rightarrow x+3z=11,\quad 3x+2z=7$<br>$\Rightarrow x=-\frac{1}{7}, z=\frac{26}{7}$<br>or<br>$z=0\Rightarrow x-5y=11, 3x-2y=7$<br>$\Rightarrow x=1, y=-2$ | Correct strategy to find a point on $l$ | M1 |
| | | Correct position vector of point on $l$ | A1 |
| | $\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$ | Correct equation.<br>(follow through their position and direction vectors but must be "$\mathbf{r}$ = ") | A1ft |
| | | | **(5)** |
| **ALT** | $x=11+5y-3z$ | | |
| | $3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$<br>$\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$<br>Eliminate one variable | | M1 |
| | $x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$ | Obtain 2 correct expressions for one of the variables | A1 |
| | $\frac{x-1}{-\frac{4}{13}}=\frac{y+2}{\frac{7}{13}}=z$ | M1 Obtain a Cartesian equation for $l$<br>A1 Correct equation | M1A1 |
| | $\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe | Deduce a vector equation for $l$<br>Follow through their Cartesian equation | A1ft |
| | | | **(5)** |

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| **(b)** | $\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$ | Correct vector joining $P$ to $Q$ | B1 |
| | $\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$ | Attempt vector product between the direction of $l$ and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ | M1 |
| | | Correct vector | A1 |
| | $\sin\theta=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$ | Angle between $PQ$ and line $n$ | |
| | $d=\lvert\overrightarrow{PQ}\rvert\sin\theta$ | | |
| | $d=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\dfrac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$ | Fully correct method for the distance | M1 |
| | $d=\dfrac{5\sqrt{481}}{39}$ | Cao Allow equivalent **exact** forms e.g. $d=\dfrac{5\sqrt{74}}{\sqrt{234}}$ | A1 |
| | | | **(5)** |
| **ALT 1** | $\mathbf{r}_m=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n=\begin{pmatrix}3\\2\\1\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ | Vector equation for either line with their direction vector from (a) | B1ft |
| | $\overrightarrow{OP}=\begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON}=\begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$ | Uses either $P$ and the parametric form of a point on $n$ OR $Q$ and the parametric form of a point on $m$ | |
| | $\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}=0$ | M1: Forms scalar product of vector $NP$ and direction vector of $l$ and equates to zero<br>A1: Correct vectors | M1A1 |
| | $\Rightarrow\mu=\dfrac{56}{117}$ | Solves | M1 |
| | $\Rightarrow d=\sqrt{\left(-\frac{85}{117}\right)^2+\left(-\frac{290}{117}\right)^2+\left(\frac{10}{9}\right)^2}=\dfrac{5\sqrt{481}}{39}$ | Obtains the correct distance | A1 |
| | | | **(5)** |
| | **Alternative for M1A1M1** | | |
| | $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}\Rightarrow d=\sqrt{\left(-1+\frac{4}{7}\mu\right)^2+(-2-\mu)^2+\left(2-\frac{13}{7}\mu\right)^2}\Rightarrow d$ is min when $\Rightarrow\mu=\dfrac{56}{117}$<br>M1: Find $d$ in terms of a parameter<br>A1: correct expression<br>M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min $d$ | | |

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| **ALT 2** | Correct vector $PQ$ | | B1 |
| | $\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}-4\\7\\13\end{pmatrix}=\left\|\begin{matrix}1\\2\\-2\end{matrix}\right\|\left\|\begin{matrix}-4\\7\\13\end{matrix}\right\|\cos\theta$ | Forms the scalar product and attempts to evaluate the LHS | M1 |
| | $\cos\theta=\dfrac{-16}{3\sqrt{234}}$ | Correct value for $\cos\theta$ exact or decimal | A1 |
| | $d=\lvert PQ\rvert\sin\theta=3\sqrt{1-\left(\dfrac{-16}{3\sqrt{234}}\right)^2}=\dfrac{5\sqrt{74}}{\sqrt{234}}$ | M1: Correct method for the distance.<br>A1: Correct **EXACT** distance | M1A1 |
| | | | **(5)** |
| | | | **Total 10** |

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